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Applying Fractional Strategies on Number Line among Primary School Students

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ABSTRACT

The teaching and learning of fractions have been getting public attention since it is one of the most problematic topics among primary and secondary students. This study aims to investigate to what extent the primary school pupils apply fractional strategies to solve problems on number line. This study was conducted using a qualitative methodology. The data were collected from task-based clinical interviews. The subjects of this study were selected among the Year Five students in Malaysia. A total of eight students participated in this study. This study revealed three types of fractional strategies. They are (1) finding an interval in fractions on number line, (2) applying concepts of decimal and interchange with fraction, and (3) comparing values of fractions. The findings showed achieving fractions arithmetic proficiency is crucial in developing the knowledge of fraction magnitude representations on the number line.

Keywords: Fractional strategies, fractions, interval on number line, number line

INTRODUCTION

Learning of fractions is crucial for the development of mathematical understanding in elementary school (Hoon et al., 2016; Kor et al., 2019; Lamon, 1999, 2001; Niemi, 1996). The clarification of putting in fractions as a major construct in developing mathematical thinking has been mutually agreed upon by Behr et al. (1983) and Kieren (1976). Yet, it is difficult for many students since it involves conceptual thinking which can be elaborated on in a few factors. Recent studies have been focusing on conceptual thinking which explains factors for understanding fractions. The factors include inductive reasoning, explanations, justifications, the

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conception of the magnitude of fractions, representations, and connections with other concepts (Nicolaou & Pitta-Pantazi, 2011). On the other hand, it is typical for teachers in elementary schools to take a longer time compared to teaching other topics, especially when guiding students to understand fractions conceptually. Many instructional devices and tools or teaching aids have been introduced to promote students' understanding in fractions (Hardman, 2005; Moyer, 2001; Reimer & Moyer, 2015). Nevertheless, the impact which can convince educators on how these interventions or any strategies used can increase students' acquisition in learning fractions is still unresolved. Besides, it is crucial to identify students' knowledge in context since all the understanding of fractions can be measured when the learning occurs contextually. The number line is also a good context that enables the educator to assess students' acquisition of mathematics knowledge, identify their errors and misconceptions contextually (Ryan & Williams, 2007). In this study, strategies for applying knowledge of fractions in number lines were investigated. The introduction of the number line seemed to be a key pedagogical decision, as it provides a learning context for an appropriate representation.

Mathematical Knowledge Prior to Fractions

In primary school, students are introduced to the topic Whole Number. A sound understanding of this topic is important before other topics are introduced. Often there are misconceptions of this topic, but fixing this problem would not require too much time compared to other topics like fractions and decimals (Moloney & Stacey, 1997). Working within whole numbers is crucial as it contributes to the learning of other related topics such as fractions and decimals (Lortie-Forgues et al., 2015). On the other hand, making a move to study the subsequent topics of mathematics, upon completing whole numbers, compels a strong foundation not only in whole numbers but also the previous concepts namely the basic mathematical operations. In fact, while working with fractions, students rely heavily on the principles of whole numbers (Karamarkovich & Rutherford, 2019). All concepts learned and mastered as prior knowledge need to be integrated and connected to the new concepts. Thus, this would enable students to grasp new concepts easily.

Learning Fractions

Fractions are rational numbers that are presented in the form of $\frac{a}{b}$, where a is the numerator and b is the denominator. The operation of the division of the two numbers, dividing the numerator by the denominator, is performed when conducting transition from fractions to decimals. In a study on students' learning of rational numbers, Ni (2001) revealed that students gained improvement of a whole number using semantic representation but there was a constraint for the students to construct the concept of fraction equivalence. The learning of rational numbers especially fractions starts at the early age of schooling, and the learning is repeated under the same topic with different content, and it is applied in other topics throughout the primary and secondary schooling period. Yet, the achievement in rational numbers is worrying. It signifies the importance of having prior knowledge. Marshall (1993) provided guidelines for students' development of rational numbers. He focused on building on the systematic improvement of students' schemata. It was emphasized that the contextual environment based on assessment helps the construction of rational knowledge. The schema is constructed from route knowledge, relational knowledge with the ability to visualize. The schemata include part-whole, quotient, measure, ratio, and operator. Hence, for more understanding of fractions, students need more experience to integrate the schemata of fractions. A number line is suggested as a representation in applying the schemata of learning (Barbieri et al., 2020; Larson, 1980; Morano et al., 2019). On the other hand, the transition of writing decimals to fractions involves both operations of multiplication and division of numbers in ten, hundred, or thousand. This transition is considered successful when the conversion of fractions to decimals and vice versa produces equal value. It was highlighted that students tended to use the decimal form when comparing the magnitude of fractions. Nevertheless, there are possibilities for mistakes made among students (Sackur-Grisvard & Leonard, 1985). Among the mistakes, students tend to interpret that $\frac{1}{3}$, 0.3 or 1.3 provide the same result. Many researchers illustrated the importance of the connection between fractions and subconstructs of fractions such as decimals (Moloney & Stacey, 1997). It was revealed that students tended to use fractions and decimals interchangeably to make the required connections when they were comparing the magnitudes (Ryan & Williams, 2007). Students who can show strong links to any alternatives of fractions have most likely achieved good knowledge in mathematics.

Applying Number Line in Fractions

It was stated that there were advantages to teach number lines for the learning of fractions since a few skills of mathematics could be integrated using number lines (Petitto, 1990). The skills include comparing numbers, sequence, finding the difference, operation, transition of numbers in the number line. The integration of fractions into number lines becomes an important component in mathematics examination (Slyke, 2019). Besides, number line offers learning opportunities for students to estimate numbers on location. Fazio et al. (2016) found that the number line platform of learning was a proper context for students to make estimation value of fractions. Since the number line was recognized as a useful representation to enhance the estimation magnitude of the fraction skill, teachers must understand to what extent their students demonstrated abilities in developing their fraction skills using a number line (Morano et al., 2019). The enhancement of mathematics skills applying the number line context increases students' mathematical concepts and hence develops their mathematical thinking. Thinking needs more passion, attention, and desire which involves conscious awareness and willingness to learn. Applying the number line is also suggested for purpose of increasing students' attention since more discussion and strategies can be demonstrated in this representation (Barbieri et al., 2020; Larson, 1980; Morano et al., 2019).

METHODS

This study was conducted using a qualitative methodology. The data were collected from task-based clinical interviews. The interviews allow an analysis of students' reasoning (Ginsburg, 1997). The subjects of this study were selected among the Year Five students in Malaysia. A total of eight students participated in this study. The students had been taught the fundamental knowledge of rational numbers in the form of fractions and decimals. Based on the learning contents in the curriculum, these students had been introduced to the relationship between fractions and decimals. The sample was chosen based on their agreement on the invitation to participate in the interview. In addition, at this age, they are able to explain and provide reasoning related to rational numbers as prompted by researchers (Moss & Case, 1999; Steffe & Olive, 2010). Each respondent was given the same question (Figure 1) and was given some time to work out the answer.

Question: *The following diagram shows an incomplete number line.*

What is the value of P?

Figure 1. Question

Then the interviewer asked questions to probe into the strategies used to derive the answer. Generally, the questions were similar, asking the respondents to justify. However, when the respondent had no clue how to find the answer, the interviewer guided by making the student more aware of the information given in the number line and linking to the prior knowledge to start working out the answer.

RESULTS

Integrating knowledge from different topics: Finding a Difference and an Interval

Overall, there is a possibility the pupils were distracted when solving the problems in fractions just for a reason of not being well prepared in managing data in terms of fraction. They might have the idea of finding a difference in an arithmetic operation, but the concept of getting a difference through intervals was not well applied in the topic of fractions. Respondent 1 (R1) could not start answering question 1 as there was no idea of what the interval meant.

I : Let’s look at this question. How do we find the value of P?
R1 : (silence)
I : In between two numbers, there is an interval. How do you find the interval?
R1 : (silence)
I : How do you find the difference between these 10 and 20?
R1 : I have to minus
I : Yes, it is good. An interval also means difference. So in between these two numbers, $\frac{1}{2}$ and $\frac{2}{5}$, what is the difference?
R1 : $\frac{1}{10}$

Similarly, R2 and R5 also had a similar problem to connect the knowledge of finding a difference and finding an interval. They did not respond to the answer but continued to answer the question when they managed to find the value of the difference. Their actions are described as in Figure 2.

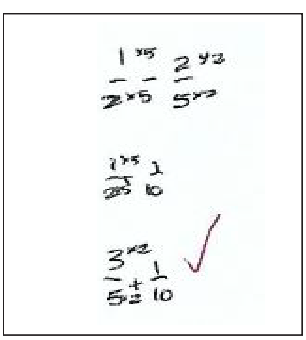

Figure 2. R2’s work

I : For the first question, what is the answer?
R2 : I don’t know how to find this answer.
I : Well, from this question can you tell me the difference between $\frac{2}{5}$ and $\frac{1}{2}$?
R2 : The difference between these two number is $\frac{1}{10}$.
I : How do you get that difference?
R2 : First I compute the denominator of these two fractions and then I subtract the fractions to find the difference.
I : Alright, now you see, what is the next step to get P value?
R2 : Before getting the P value I add $\frac{1}{2}$ with $\frac{1}{10}$ and get $\frac{3}{5}$. After that, I add $\frac{3}{5}$ with $\frac{1}{10}$ and get $\frac{7}{10}$.

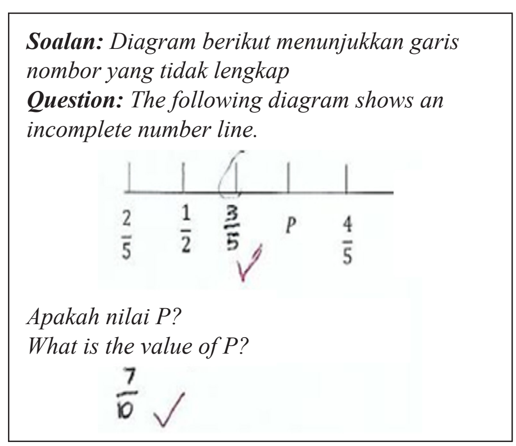

Figure 3. R2’s answer

I : Are you sure that is the answer for P value?
R2 : Yes, I am sure.

I : Why do you say that?

R2 : Because I try to add $\frac{7}{10}$ with $\frac{1}{10}$ and get $\frac{8}{10}$ which is equal to $\frac{4}{5}$ when simplified (as shown in Figure 3)

I : Very good.

R5 responded in a similar way as R2, as described below.

I : How do you find this question? Please find the answer.

R5 : I am not sure how to answer this question.

I : Well, what is the difference between $\frac{2}{5}$ and $\frac{1}{2}$?

R5 : (silence)

I : Well, how about you make the determinant of all the fraction into the same value.

R5 : (silence)

I : When you want to make the determinant the same. You need to get the same values of denominators for fraction $\frac{2}{5}$ and fraction $\frac{1}{2}$.

R5 : I got $\frac{4}{10}$ and $\frac{5}{10}$.

I : Yes, now what is the difference between these two values?

R5 : $\frac{1}{10}$

I : Well done. So, what is the value of P?

R5 : $\frac{7}{10}$

But, it was different with R4. R4 managed to answer the question by interpreting the line number as a sequence. R4 was familiar with this type of question and said that it was by getting the difference. R4 also shared the concepts in terms of a sequence of increasing and decreasing. R4 had sound knowledge of how to manage the values of fraction as expressed in Figure 4.

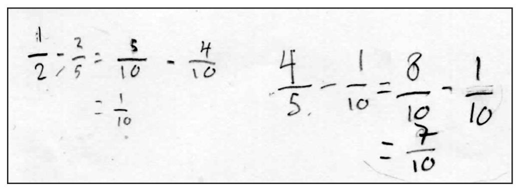

Figure 4. R4's work

I : As you already know, these are mathematics subject questions. Have you seen these kinds of questions before? (Referring to number line)

R4 : Yes.

I : So, have you learned this in school?

R4 : Yes.

I : So from your working here, can I ask why are you subtracting $\frac{2}{5}$ from $\frac{1}{2}$? Why do you subtract instead of adding?

R4 : Because these numbers here are going up

I : So you notice that this fraction $\frac{2}{5}$ is bigger than $\frac{1}{2}$?

R4 : Yes

I : Good.

Applying Decimal Concept to Answer Fractions

Some respondents found that it was easier to answer fractions questions using fractions. The pupils who have knowledge of fractions and decimals seem to prefer presenting the comparison of numbers in decimals. This situation was shown by R3.

I : Can you show to me the answer?
R3 : Yes, can.

R3 drew a line first and copied the information from the question. Below each fraction, R3 wrote the decimal that represented the fraction (0.4, 0.5, and 0.8 representing $\frac{2}{5}$, $\frac{1}{2}$ and $\frac{4}{5}$ respectively) as shown in Figure 5. Then wrote 0.6 on the missing part of the number line, between $\frac{1}{2}$ and *P*. Finally wrote 0.7 below *P*.

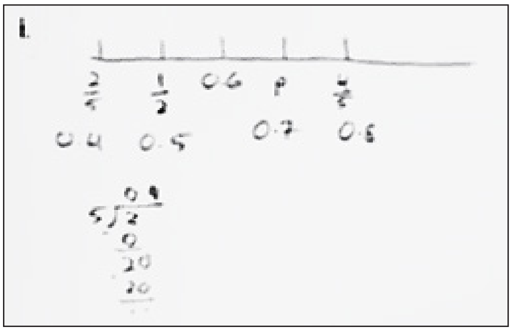

Figure 5. R3's work

I : Can u change the number 0.7 into a fraction form?
R3 : Yes, Can. and $\frac{7}{10}$. (said verbally)
I : Can u show me how do you change $\frac{2}{5}$ to get 0.4?
R3 : (showed it – the working using long division below the number line)

Comparing Values of Fractions with Same Denominators

Pupils need to have the specific knowledge to manage the learning of fractions. It is important for pupils to develop knowledge of comparing values of different types of numbers. There are concepts to be learned in comparing different types of numbers. In this study, R6 and R7 applied the concept of getting the same denominators in comparing the fractions. They held the major concepts of getting the same denominators, they also possessed knowledge of getting a value for each interval and numbers in a sequence. R6 shared the way to get the answer as below.

R6 : Wait, give me time. Why the bottom value is different? I think I can. Is it $\frac{7}{10}$?
I : Yes, it is $\frac{7}{10}$. May I know how you solve this even the denominator?
R6 : I change the fraction to the same denominators. I memorise the method conversion of $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{2}$. Then, I just times the number on top after getting the same number at the bottom, I mean the same denominators. I can see each number increases by $\frac{1}{10}$.

Similarly, R7 shared that the difference of the values after changing all the denominators to a common denominator. R7's description is presented below.

R7 : It is $\frac{7}{10}$.

I : How did you get the answer?

R7 : I need to find the same denominator for both $\frac{2}{5}$ and $\frac{1}{2}$. I can use 10 as the denominator. So, $\frac{2}{5}$ equal to $\frac{4}{10}$ while $\frac{1}{2}$ equal to $\frac{5}{10}$. Then the difference between $\frac{2}{5}$ and $\frac{1}{2}$ is $\frac{1}{10}$.

Pupils may have difficulties in solving fractions if they have less knowledge in dealing with the lowest common multiple (LCM). Hence, pupils need to have a strong background of knowledge in the topics which provide the prior knowledge for learning fractions. In learning fractions, a conceptual understanding of getting a common denominator is important. Without the skill, they may have difficulties finding a fraction number or comparing numbers as well as working on the operations of numbers. R8 showed this problem. There was lesser readiness to solve the problem in fractions. It was noticed that the confidence level was lower especially to find the common denominators for the fractions in the sequence as expressed below.

I : What is the first thing you do when you answer this question?

R8 : $\frac{1}{2}$ times 5

I : Why?

R8 : Because the other denominator also 5.

I : So what is your new denominator for $\frac{1}{2}$?

R8 : 10

But, R8 could not find the LCM of 2 and 5 consistently because '5' was multiplied to the denominators of $\frac{2}{5}$ and $\frac{4}{5}$, as what was done for $\frac{1}{2}$, causing confusion that resulted in the constraints of getting the answer. R8 only managed to proceed with the work after getting some assistance.

I : But the denominators for $\frac{2}{5}$ and $\frac{4}{5}$ are still 5.

R8 : Then I will multiply with 5 also.

I : So what did you get for the new denominator for $\frac{2}{5}$ and $\frac{4}{5}$?

R8 : 25

I : So did you realize all three denominators are not the same even after you changed them?

R8 : (silence)

I : Okay. If you found a question like this what is the first thing you need to do?

R8 : See the denominator.

I : So our case now all denominators all different. Then we need to make them the same.

R8 : Right.

DISCUSSION

The three meaningful findings that have surfaced from this study are: (1) finding an interval in fractions on a number line, (2) applying concepts of decimal and interchange with fraction, and (3) comparing values of fractions.

Finding an interval or in other words, finding the difference between two numbers is common mathematical knowledge by applying subtraction. In this study, it was found that the pupil who was not confident to conduct an operation to find the interval failed to get a number on the number line. Even though the method of finding a difference or interval between two numbers was introduced at the beginning of the year of their study they still have difficulty in applying the concept. Hence, the inadequacy of this knowledge caused the disability to interpret the concept of interval in fraction This type of knowledge is not only seen when working on a number line or fractions but also in many other contexts of mathematics such as finding the difference between decimals, whole numbers or other non-routine mathematics problems (Namkung et al., 2018). In many different contexts or situations of applying concepts of finding the difference requires more knowledge than merely using the operation of subtraction. Their acquisition of knowledge in the application of subtraction operation can be seen in finding the distance between two points like finding a gap of ages. If the students managed to interpret and express the meaning for the steps of finding the difference, they will be able to show the ability to find any unknown in the number line. Nevertheless, many researchers express that students need more exposure and guidance in getting into a clear picture for the acquisition of mathematics knowledge especially learning fractions (Bossé et al., 2019). The main aim of the instruction includes enhancing students' acquisition of mathematics knowledge through inquiry. More questioning can be implemented with the application of graphical representation. Graphical representations such as number line are used to relate to pupils' understanding of learning. Students' frequent effort of using number line would help them to explain and provide reasoning in getting and completing the sequence of numbers located on the number line (Tunç-Pekkan, 2015). Hence, conceptually understanding the relationship between finding a difference and interval needs to be enhanced in the teaching and learning when they are exposed to different numbers namely whole number, fractions, and decimal numbers.

The emphasis of similar concepts of finding the difference and interval is also practised in other topics like whole numbers, fractions, and decimals. Nevertheless, gaining a comprehensive understanding of these topics is not an easy task since the knowledge of fractions is not only difficult but is also applied in many important topics of mathematics (Lortie-Forgues et al., 2015.). In this study, the findings show that working on decimals is easier than working on fractions. Many researchers agreed that learning fractions is more difficult compared to other topics such as decimals and percentages (DeWolf et al., 2015; Hurst & Cordes 2016; Zhang et al., 2013). For this reason, it is certain that proficiency in fractions can help pupils to get a better understanding of multiple topics in mathematics. For a full understanding of fractions, a student should be flexible enough to apply fractions and decimals and the translation between these two. Although decimals and fractions are taught to all students in primary school, the development of knowledge for the two topics has been reported as inadequate among secondary and even some tertiary students in many countries (Brown, 1981; Nesher & Peled, 1986). Only, those students who are able to translate between fractions and decimals, manage to solve multiple problems in comparing the magnitude of numbers in these two forms. This finding echoes the outcome of Ryan and Willians (2007). They found that students who possessed good knowledge of fraction–decimal equivalents were able to make good use of it in the context of number line. The findings also showed achieving fractions arithmetic proficiency is crucial to develop the knowledge of fraction magnitude representations. In this study, for comparing fraction magnitude or value, the pupils needed to find the difference between two fractions. In the process, the Lowest Common Multiple concepts had to be applied for getting an equivalence in the denominator. They applied the knowledge of multiplication in the steps. Findings in this study support outcome of research presented by Siegler et al. (2011) who interpreted that developing procedural knowledge in fraction arithmetic was vital for students to carry out other aspects of strategies in solving mathematics problems. It echoes that fraction arithmetic can be taught as a support skill when other strategies are applied such as finding an interval between fractions in a number line.

This study provides more input on how students work on a number line. It opens more ideas for teachers to observe students' ability to apply fraction knowledge specifically and how they deal with related knowledge in handling the specific problem on a number line. The input is essential for improving students' learning since learning fractions can be difficult and confusing when the fundamentals are not mastered. In addition, the content of mathematics becomes more compact. It is reasonable to learn fractions together with other rational knowledge namely decimals and percentages with the reality that fractions encompass all of the ways of expressing rational numbers (Fazio & Siegler, 2011).

CONCLUSION

This study has an impact on some focused strategies in learning rational numbers (fractions and decimals). The development of knowledge of representation in rational numbers can be emphasised in two major aspects. Firstly, conducting fraction operations is a major skill in the process of representation of rational numbers. Students' competency in comparing values of fractions may be influenced by their proficiency in conducting the operation of the fraction. Getting the same denominator is part of the procedure in the operation. Secondly, finding differences in a number line increases the attainment of magnitude knowledge in rational numbers. Within the knowledge of rational numbers (decimals and fractions), the decimal is a subconstruct of fraction since changes from decimals to fraction can be easily done by defining the place values taken, and consideration of writing into tenths, hundredths, and more. Nevertheless, the transition from fraction into decimal involves a more complicated process (Sackur-Grisvard & Leonard, 1985). Thus, some students may find it difficult to make this kind of transition. Failing to do so, they are not able to compare the values in decimals. Ideally, comparing the rational values in terms of decimals may be easier compared to the form of fraction. Nevertheless, when a number line is used in the comparison for a specific arithmetic sequence, it helps to increase students' understanding (Morano et al., 2019; Slyke, 2019). The understanding can be observed by adding in the concept of difference to get the subsequent fraction in the number line, according to the findings from the interview.

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